

Multiplication Basis of Emergence of Classifiers^{*}

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This paper aims to develop a multiplication-based inquiry into the emergence of classifiers by answering three questions. What is the role of the nominal suffix (Kayne 2005) in the multiplicative representation of the whole number? How is the nominal suffix linked to the emergence of classifiers? How do classifiers emerge as parcelers and dividers via multiplication (Landman 2004, Borer 2005)? With the assumption that the coding of the multiplicative operation in language is necessary, it is suggested the multiplicative identity, one, in multiplication plays a major role in deriving the nominal suffix and hence the features of classifiers. Finally, this proposal is used to highlight the design of the number faculty by examining the set-theoretic definition of natural numbers.

Key words: classifier, emergence, multiplication, number

1. Introduction

This paper develops an interface inquiry into the emergence of classifiers in language through a nominal affix attaching to the multiple portions of numbers. The emergence of classifiers aims to answer why language needs classifiers to serve the functions of counting, individuating, grouping, and so on. In order to understand some underlying reason, we need to know how the category emerges. How a grammatical category comes into being can be studied through the historical development of a language which gives us the point of origin for the category. We can also document the language development of a child and see how the category is acquired in his or her grammar. The emergence of the category in one family of languages but not in others can be approached by investigating how the family with the category gradually transfers the features of the category to the one without through language contact. The perspective adopted in this paper is to look at how the classifier emerges in language as a result of the interaction between the number faculty and the language faculty. If the number

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faculty sends an instruction to the language faculty that multiplication needs to be linguistically encoded, then the language faculty has to scrutinize the conditions and constraints of this mathematical operation for the linguistic design and finally produces an optimal output. Call it the classifier. If this emerging process through multiplication is a virtual necessity for every language, then the classifier should be a universal product of this number-language crossover.

So far, the literature does not seem to have taken a clear position as to whether the classifier is universal or language-specific. After Determiner Phrase had been proposed in Abney (1987) as a functional category of nominals in western languages such as English, studies of universal grammar started to apply the Determiner Phrase proposal to eastern languages and suggested that Chinese for instance also has Determiner Phrase in nominals as in the pioneering work of Tang (1990). Another nominal element, Number/Numeral Phrase has followed a similar line in recent decades, and its postulation for Romance and Germanic languages (e.g. Ritter 1991) was subsequently applied to the nominal structure of Chinese (e.g. Li 1998, 1999 and Cheng & Sybesma 1999). The importation of the universal treatment of these nominal elements from the West to the East has by now gathered sufficient momentum to develop into a strong line of research within generative grammar.

However, another nominal element, the classifier, does not seem to enjoy comparable spreading from classifier languages to non-classifier ones. One way to actualize this inquiry is to ask whether the classifier also occupies a phrase level in the syntactic tree of nominals in English as in Chinese. Although typological studies have long observed some measuring or classifying construction in English such as *a cup of tea*, there is still little attention to the formalization of such a classifier category in the syntax of English. Nor did the research of nominals in Chinese and other classifier languages lead to any concrete implication of its structural status in non-classifier languages (e.g. Simpson 2005, Cheng & Sybesma 2005, Tang 2005).

In principle, there was no difficulty in suggesting that the classifier be a universal category deserving its own level in the syntax of all languages. This suggestion could be strengthened by the complementary distribution of classifier and plural inflection in language (T'sou 1976). The fact that everyone needs to count would lend further support to the classifier as a tool for counting and hence its universal necessity in language. The way to implement the classifier proposal in English, for example, would be more or less the same way in which the Determiner Phrase model has been applied in Chinese. However, viewing nominals in western languages through a classifier lens lags as far behind viewing eastern languages through the lenses of Determiner Phrase and Number/Numeral Phrase.

A solution to the classifier problem began showing promise in the recent studies of

Landman (2004) and Borer (2005). They provided a semantic and a syntactic account of how the classifier individuates, suggesting that it should also exist in English as a parceler or a divider. Their detailed proposals stand out in the literature where it is generally assumed that the category simply divides nouns into discrete units for counting without any elaboration of how such individuation is formalized (e.g. Allan 1977, Paris 1989, Croft 1994, Cheng & Sybesma 2005).

While the aforementioned authors attempted to apply a universal grammar approach to the classifier system, this paper aims to unite the parceling and dividing proposals by means of the multiplicative operation in the number faculty. If such multiplication-based interface investigation of the classifier should prove successful, the category can be confidently argued to exist in the syntax of all languages.

But how is the classifier as an individuating device explained via multiplication? Such a mathematical link can be exemplified by the idea of grouping in the following three-tier structure. Consider:

oooo			
oooo	oooo	oooo	oooo
oooo	oooo	oooo	oooo
oooo	oooo	oooo	oooo

According to Lamon (1994:92), when “one reframes a situation in terms of a more collective unit,” one “invokes a part-whole schema.” The first row presents sixteen objects as sixteen units. We then have sixteen one-units. The second row gives units of units, i.e. four composite units, each of four one-units. We then have four four-units. The third row creates units of units of units, giving rise to one three-unit consisting of three of the four four-units.

While this grouping combination within the multiplicative structure is easy to associate with the idea of classifiers as an individuating or a grouping device, the use of the multiplicative operation was found in a recent manuscript by Kayne (2005) to explain the linguistic coding of numbers. Basing on a bound morpheme following the ten-based multiple components in a number in French for example, he proposed that the morpheme is a nominal suffix attaching to these multiple components and the suffix is essential for the multiplication in a number in all languages. For instance, Number 600 has the surface structure *six hundred*, but with the underlying structure is *six hundred-Nominal Suffix* where this suffix is silent in English but overt in French. The study brings an edge to the universal perspective of this nominal suffix motivated by the need to meet the conditions of multiplication.

It has been reported that language and number are closely related to each other (Hurford 1987). One salient point that the two faculties share is the property of discrete infinity (Chomsky 1988).¹ Sentences are meant to grow to an unlimited length while number can increase to as great a number as we wish. In the process of growth, a sentence lengthens by a discrete unit. So there can be five words in a sentence, but never be five point six words. The same discreteness applies to the growth of a number. So in the system of natural numbers, a number increases by adding one but not by one point two. Chomsky further explicates the closeness between the number and the language faculty as the derivation of the former from the latter as follows (citing from Botha 2003:58):

“The number faculty developed as a by-product of the language faculty.
The number faculty is an abstraction from human language.”

Regarding such an affinity between language and number, it is high time we should justify the emergence of classifiers from the perspective of multiplication in order to find out if the grammatical category can satisfy the interface conditions of multiplication in the number faculty.

If multiplication is so essential and important that the classifier category and the nominal suffix are the consequences of meeting the conditions of this multiplicative interface by the language faculty, then discovering how these two grammatical elements are theoretically combined will be exciting. The inquiry into the emergence of classifiers in terms of the multiplicative basis is broken down into the following questions:

- i. What is the role of the nominal suffix in the multiplicative representation of the whole number?
- ii. How is the nominal suffix linked to the emergence of classifiers?
- iii. How does the classifier emerge as a parceler and a divider via multiplication?

2. Background

2.1 Classifiers for parceling

Although in the literature it has long been accepted that the classifier is used to singularize nouns into individuals for counting, there is little elaboration of what is meant by such individuation. For example, although Cheng & Sybesma (2005:276) generalized the views of the classifier as ‘singling out one entity from the plurality of

¹ Another common property of language and number is the idea of recursion or embedding (Hauser, Chomsky & Fitch 2002).

entities provided by the semantic representation of the noun in the lexicon; it picks out one instance of what is denoted by N', they did not show how the classifier performs the picking-out. Recent research by Landman (2004) and Borer (2005) demonstrated how it works.

Landman (2004) viewed the classifier as a parceling device that divides a set of non-countable sums into a set of countable parcels, since sums cannot be counted directly. After parceling, the noun phrase following the classifier then fits into another semantic domain for interpretation. Consider the classifier-*of* construction in English.

English takes the particle-*of* in the classifier construction:

- (1) Three bottles of water [from mass to count]
- (2) Three groups of boys [from sums to groups]

In *three bottles of water*, the mass entity, *water*, is parceled by the classifier, *bottle*, to fit into bottles which are the parcels for counting. In *three groups of boys*, sums of boys are parceled by the classifier, *group*, so that the counting is done through these groups.

The author proceeded to argue that *time* in English (as well as in Dutch) is also a classifier performing the parceling function. Basing his argument on the agreement contrast between the noun and *keer* (time) as follows:

- (3) *Drie jongen / Drie jongens
 Three boy / Three boys
 'Three boys'
- (4) Dafna sprong drie keer/keren
 Dafna jumped three time/times
 'Dafna jumped three times.'

he concluded that *keer* in Dutch behaves like a classifier, which can optionally agree with its preceding number, unlike a noun which requires necessary agreement with the number.²

Having verified the classifier status of *time*, he pointed out that one of the readings of the following sentence is due to the parceling function of *time*. Consider:

² This paper treats numbers and numerals, as well as Number Phrase and Numeral Phrase, as equivalent since major argument on number agreement or the singular – plural contrast among these will not be covered here.

- (5) Four times three boys met in the park.
[Reading 1: A group of twelve boys....]
[Reading 2: Four groups of three boys.....]

Reading 1 is obtained by the multiplicative operation between *four* and *three* with the product of twelve. Reading 2 is yielded by the parceling function of *time*, which parcels the interpretation of *three boys*, (i.e. the set of sums of boys of the cardinality of three), into appropriate parcels. These parcels become countable and are counted by *four*.³

Although Landman was interested in the reading of ‘four groups of three boys’ rather than ‘a group of twelve boys’, can we also derive a similar expression for the latter such as *one times twelve boys*? If so, the question is how the classifier *time* (or the classifier in general), as a parceler, is explained in terms of multiplication.

Bearing this classifier-multiplication question in mind, how can we fit the expression *four times three boys* into the syntax of number expressions as the Number-Classifier-Noun sequence?

2.2 Classifiers for dividing

While Landman’s semantic account for the classifier does not touch upon how *time* as a parceler is accommodated in the Number-Classifier-Noun sequence in classifier languages, this brings us to Borer’s (2005) view of the classifier as a divider within a similar nominal construction, which promotes a structural explanation of count-mass distinction.

In the light of the arbitrary count-mass distinction due to the following data:

- (6) A wine/wines, a love/loves, a salt/salts (on count reading)
(7) There is dog/stone/chicken on this floor (on mass reading)

Borer extended Chierchia’s (1998) view that all nouns in Chinese are mass to the universal view that all nouns in all languages are mass.

It was suggested that nouns in mass state be apportioned by a number of dividers. Relevant to this paper are the projection of classifiers in Chinese and the use of plural inflection in English. These two dividers mould nouns into countable units to yield the count interpretation. On the contrary, the absence of these dividers leaves nouns to be interpreted as mass. Therefore, what make a noun countable are its structural features rather than its lexical properties.

³ Interested reader can refer to Landman (2004:243) for the formal semantic representation of *time*.

For instance, as there are no dividers in the syntax of examples (8)-(9), the interpretation of the noun *yan/salt* is mass. When the divider is realized as the classifier, *zhi*, in (10) and as the plural *-s* in (11), the noun *mao/cat* receives the count interpretation specified by the number *san/three*. Comparable to the classifier item in Chinese, English uses plural inflection as a divider to portion mass-CAT into count-*cats* with the bound morpheme *-s* realized in Divider Phrase. That is why the plural inflection and the classifier are complementarily distributed, at least in English and Chinese (T'sou 1976).

- | | | | | | | | | |
|------|--------|---------|--------|----------|-------|-------|----------------|-----|
| (8) | Mass: | [Number | haoduo | [Divider | | [Noun | yan |]]] |
| (9) | Mass: | [Number | much | [Divider | | [Noun | salt |]]] |
| (10) | Count: | [Number | san | [Divider | zhi | [Noun | mao |]]] |
| (11) | Count: | [Number | three | [Divider | cat-s | [Noun | cat |]]] |

While the literature on the syntax of classifiers assumes the Number-Classifier-Noun order with the consideration of the surface form in nominals, Borer started from the ontological need of a dividing device as universally required by language and proceeded to argue for a syntactic slot for it. If the classifier as a divider is really located in the Number-Divider-Noun structure, and since the classifier, *time*, in Landman's proposal is related to the multiplicative operation, it is then natural to ask how the classifier in the Number-Divider-Noun construction is explained in terms of multiplication? Can multiplication in the end explain the emergence of classifiers both as a semantic parceler in Landman (2004) and as a syntactic divider in Borer (2005)?

In this connection, we can continue to ask: (i) how the multiplication expression *four times three boys* in Landman's study is accommodated in Borer's structure (Could it be [Number *four*] [Divider *times*] [*three* ?] [Noun *boys*]? While *time* is argued to be a classifier and stay in Divider, what about *three*?); and (ii) how can we further enrich the understanding of the plural *-s* in terms of the multiplicative account, given that *twelve boys* was argued to realize the plural inflection *-s* as in [Number *twelve*] [Divider *-s*] [Noun *boy*]?

The above two studies have clearly opened up an area on the global status of the classifier as a parceler or a divider in language. Despite the different approaches towards the universality of the grammatical category, what sounds promising is the direction of a multiplicative explanation to unite them. Let us move on and look at how the multiplication of numbers interacts with language.

2.3 Nominal suffix for multiplication

Kayne (2005) tackled this problem by investigating how Number-Multiple expressions such as *three hundred* are expressed in English, French, and Romanian, and generalized

that a nominal suffix is attached on the multiples in both Approximate-Multiples and Number-Multiples.

Regarding Approximate-Multiples, he found that an overt nominal suffix *-aine* is present in French and suggested a silent *-AINE* in English, as in:

Approximate-Multiples:

- (French)
- (12) Cent-aine (a hundred)
- (English)
- (13) Hundreds of books ← Hundred-AINE-s of books
- (English)
- (14) Tens of thousands of books ← Ten-AINE-s of thousand-AINE-s of books

Through the comparison of similar properties between Approximate-Multiples and Number-Multiples in French and English, the nominal suffix was also suggested for Number-Multiples. Data were drawn from the interaction with the quantifier *several/plusieurs*. Consider:

Approximate-Multiples:

- (15) Plusieurs centaines de dollars (French)
- (16) Plusieurs milliers de dollars (French)
- (17) *Several hundreds of dollars (English)
- (18) *Several thousands of dollars (English)

Number-Multiples:

- (19) *Plusieurs cent dollars (French)
- (20) *Plusieurs mille dollars (French)
- (21) Several hundred dollars (English)
- (22) Several thousand dollars (English)

On the one hand, within the French dataset, the presence of *plusieurs* is sensitive to the suffix on both types of multiples, i.e. grammatical in (15)-(16) and ungrammatical in (19)-(20). But on the other hand, Approximate-Multiples in French in (15)-(16) look similar to Number-Multiples in English in (21)-(22) in the presence of the quantifier. Because of this similarity as well as the requirement for the suffix in Approximate-Multiples in French, a similar silent suffix, *-AINE*, was also suggested for Number-Multiples in English, as in:

- (23) Several hundred-AINE dollars
- (24) Several thousand-AINE dollars

By now, the nominal suffix in French and English is required to attach on both Approximate-Multiples and Number-Multiples where the grammatical variation lies in the overtiness of the suffix.

What is this nominal suffix used for in both types of multiples? Kayne based his argument on similar structures in Romanian and proposed that the suffix should be for the purpose of multiplication between digits and their respective multiples in a number. In Romanian, multiples look like their English counterparts in that they do not take a preposition (25)-(26).

- (25) Ten (*of) books (English)
- (26) Zece (*de) books (Romanian)
- Ten of book
- ‘Ten books’

However, when the number in Romanian is multiplied by *trei* (three) to make up *treizeci* (thirty), the suffix *-i* as well as the preposition is required (27).

- (27) Trei-zec-i de books
- Three-ten-Suffix of book
- ‘Thirty books’

At the end of the study, the author claimed that the nominal suffix may be equivalent to an unpronounced SET in the following:

- (28) Threes/ groups of three/ sets of three

But he did not go on indicating how the suffix interacts with the single-digit *three* or with the idea of SET.

No matter whether this nominal suffix is overtly or covertly realized on multiples in language, Kayne considered that it should be a requirement for spelling out the Number 300, for instance. Since the spell-out as *three hundred* in English and *san bai* (three hundred) in Chinese is based on the multiplication between the digit *three/san* and the multiple *hundred/bai*, what is the role of the nominal suffix on *hundred/bai* in the multiplicative representation of the whole number? Given the significance of the nominal suffix on the multiple component of numbers, can we do the same for the single-digit *three* by its being attached with the nominal suffix since *three* can be expressed in

terms of the power of ten as 3×10^0 (three), just like numbers with other multiples as 3×10^1 (thirty), 3×10^2 (three hundred), and 3×10^3 (three thousand)?

In his sense, the nominal suffix is a universal grammatical morpheme for the purpose of multiplication in a number. Shortly before, we have come up with a question of how multiplication can explain the emergence of classifiers as a parceler and a divider. The next question to ask is how the nominal suffix under the multiplicative framework is linked to the origin of classifiers.

In the rest of this paper, along the number-language interface journey from multiplication of numbers, through the nominal suffix, and to the emergence of classifiers, the research questions are going to be answered in §3. Section 4 predicts the implication of the relation between classifiers and the definition of natural numbers, followed by the conclusion in §5. The framework is outlined in Table 1.

Table 1: The framework

Number-language interface	Questions
Multiplication of numbers ↓	1. What is the role of the nominal suffix in the multiplicative representation of the whole number?
Nominal suffix ↓	2. How is the nominal suffix linked to the emergence of classifiers?
Emergence of classifiers	3. How does the classifier emerge as a parceler and as a divider in terms of multiplication?

3. Classifier as multiplication

3.1 Multiplication in numbers and groupings

When we call a number, the calling sequence reflects a multiplicative relation. For example, we do not normally pronounce the Number 666 as *liu-liu-liu* in Chinese or as *six-six-six* in English. Instead, we use *liu-bai liu-shi liu* or *six hundred and sixty-six*. What we say is different from the linear sequence of the digits in the number.

$$\begin{aligned}
 \text{Since } 666 &= 6 \times 100 + 6 \times 10 + 6 \times 1 - (a) \\
 &= 100 \times 6 + 10 \times 6 + 1 \times 6 - (b) \\
 &\quad (\text{by Commutative Law for multiplication: } m \times n = n \times m),^4
 \end{aligned}$$

then our calling sequence is based on the Digit-Multiple order in (a) rather than the

⁴ Despite the same final product of these two representations, they are significantly different in the coordinate system in geometry.

Multiple-Digit order in (b). In Chinese, *liu-bai* (six hundred) in *liu bai liu shi liu*, corresponds to 6×100 , *liu-shi* (six ten) to 6×10 and *liu* (six) to 6×1 in (a). The components in (b) do not reflect such a calling sequence. Similarly, for *six hundred and sixty-six* in English, *six hundred* maps to 6×100 , *sixty* to 6×10 and *six* to 6×1 in (a).

The same holds for numbers ending with the digit zero.

For $600 = 6 \times 100 = 100 \times 6$, the calling order is *liu-bai/six hundred* which fits with the multiplicative representation as 6×100 rather than 100×6 .

For $60 = 6 \times 10 = 10 \times 6$, the calling order is *liu-shi/sixty* which fits with the representation as 6×10 .

For $6 = 6 \times 1 = 1 \times 6$, although *liu/six* does not apparently show which representation is employed in its calling, by analogy with the above cases, the representation is assumed to be 6×1 instead of 1×6 .⁵

To summarize, for numbers 600, 60, and 6, the multiplicative representations used in calling are 6×100 , 6×10 , and 6×1 respectively.

If we allow adjacent combination without altering the order of members by the Associative Law for multiplication, $(a \times b) \times c = a \times (b \times c)$, those in Column A of Table 2 can change to those in Column B and then to Column C.⁶ The original multiple on the right side of the multiplicative operator, ' \times ', decreases to one while the digit on the left side proportionally increases. This rearrangement is the same for 666. Since representations in Column A reflect our calling sequence, those in Column C do it, too. The final representations in Column C unanimously show that for all the sample numbers, the $\times 1$ element is extracted and stays at the end of each multiplicative representation and remains silent in the process of number calling. In general, the calling of Number m is based on the $m \times 1$ order with the silent $\times 1$ element rather than the $1 \times m$ order.

⁵ Here is an asymmetry. For 6×1 , we just code 6 without the $\times 1$ portion as *liu/six* instead of **liu-yi/*six-one*, unlike the other multiplicative representations. We will come back to this point later.

⁶ The rearrangement of elements in multiplication due to Associative Law resembles Chomsky's (2005:11) idea that 'operations forming complex expressions should consist of no more than a rearrangement of the objects to which they apply, not modifying them internally by deletion or insertion of new elements.' In other words, some behaviour in multiplication is similar to comparable operation in language.

Table 2: Multiplicative representations of number calling

Number	A	B	C
600	6×100	60×10	600×1
60	6×10	60×1	60×1
6	6×1	6×1	6×1
666	$6 \times 100 + 6 \times 10 + 6 \times 1$	$600 \times 1 + 60 \times 1 + 6 \times 1$	666×1
m	$m \times 1$		

Since this silent $\times 1$ element comes from various multiple portions in a number and recall that it is suggested that Kayne's nominal suffix be attached onto each multiple portion of a number for the purpose of multiplication, then it is very tempting to correlate this $\times 1$ element with the nominal suffix one way or another.⁷

In the light of this promising marriage of the silent $\times 1$ element and the nominal suffix proposal, let us move on to the other function of this $\times 1$ in the multiplicative representation in order to shed more light not only on the property of the nominal suffix, but also on the implication of the internal grouping inside a number for the emergence of classifiers.

Let us look at the function of this $\times 1$ element inside a number. Consider Number 666 in the following four internal groupings.

Case I:

$$\begin{aligned}
 666 &= (111+111) + (111+111) + (111+111) - (a) \\
 &= [(111+111) + (111+111) + (111+111)] \times 1 \text{ (since } m = m \times 1) \\
 &= (111+111) \times 1 + (111+111) \times 1 + (111+111) \times 1 \text{ (by Distributive Law for multiplication)} \\
 &= 2 \times 111 \times 1 + 2 \times 111 \times 1 + 2 \times 111 \times 1 \text{ (since two tokens of 111 in each bracket pair)} \\
 &= 3 \times (2 \times 111 \times 1) - (b) \\
 &= 3 \times 2 \times 1 \times 111 - (c) \text{ (by Associative Law for multiplication)}
 \end{aligned}$$

In Case I, Number 666 can finally be represented by two multiplication forms in (b) and (c) from the addition form in (a). Let us ignore the difference between (b) and (c) for the time being. After all, these two forms correspond to the grouping in (a), i.e. three groups of two 111's each.

⁷ This line of multiplicative reasoning further motivates the nominal suffix-on-multiple proposal for restricting the verbal coding order as Digit-Multiple (e.g. *liu bai/six hundred*) rather than the opposite Multiple-Digit order (e.g. **bai liu/*hundred six*).

Case II:

$$\begin{aligned}
 666 &= (111+111+111) + (111+111+111) \text{--(a)} \\
 &= [(111+111+111) + (111+111+111)] \times 1 \text{ (since } m = m \times 1) \\
 &= (111+111+111) \times 1 + (111+111+111) \times 1 \text{ (by Distributive Law)} \\
 &= 3 \times 111 \times 1 + 3 \times 111 \times 1 \text{ (since three tokens of 111 in each bracket pair)} \\
 &= 2 \times (3 \times 111 \times 1) \text{--(b)} \\
 &= 2 \times 3 \times 1 \times 111 \text{--(c) (by Associative Law)}
 \end{aligned}$$

Another grouping of Number 666 is presented in Case (II) through the addition form in (a). Again we finally come up with two multiplication forms in (b) and (c). They correspond to the grouping in (a) as two groups of three 111's each.

Case III:

$$\begin{aligned}
 666 &= (111)+(111) + (111)+(111) + (111)+(111) \text{--(a)} \\
 &= [(111)+(111) + (111)+(111) + (111)+(111)] \times 1 \text{ (since } m = m \times 1) \\
 &= (111) \times 1 + (111) \times 1 + (111) \times 1 + (111) \times 1 + (111) \times 1 + (111) \times 1 \text{ (by Distributive Law)} \\
 &= 1 \times 111 \times 1 + 1 \times 111 \times 1 + 1 \times 111 \times 1 + 1 \times 111 \times 1 + 1 \times 111 \times 1 + 1 \times 111 \times 1 \\
 &\quad \text{(since one token of 111 in each bracket pair)} \\
 &= 6 \times (1 \times 111 \times 1) \text{--(b)} \\
 &= 6 \times 1 \times 1 \times 111 \text{--(c) (by Associative Law)}
 \end{aligned}$$

If there is only one 111 in each bracket pair, the grouping will be expressed by the addition form in (a). It depicts the grouping as six groups and each group contains one token of 111, which is finally represented by the multiplication forms in (b) and (c).

Case IV:

$$\begin{aligned}
 666 &= (111+111+111+111+111+111) \text{--(a)} \\
 &= (111+111+111+111+111+111) \times 1 \text{ (since } m = m \times 1) \\
 &= (6 \times 111) \times 1 \text{ (since six tokens of 111 in the bracket pair)} \\
 &= [1 \times (6 \times 111)] \times 1 \\
 &= 1 \times 6 \times 111 \times 1 \text{--(b)} \\
 &= 1 \times 6 \times 1 \times 111 \text{--(c) (by Associative Law)}
 \end{aligned}$$

Finally, six 111's can be viewed as forming a single, collective group in (a). The addition form in (a) is then transformed into two multiplication forms in (b) and (c).

Diagrammatically, the four groupings of Number 666 can be replaced by different packs of Symbol 'o' and denoted by an addition form as well as two multiplication forms in Table 3.

Table 3: Relation among grouping, addition, and multiplication

666		
Grouping	Addition	Multiplication
I oo oo oo	$(111+111) + (111+111) + (111+111)$ (where o = 111)	$3 \times 2 \times 111 \times I$ $3 \times 2 \times I \times 111$
II ooo ooo	$(111+111+111) + (111+111+111)$	$2 \times 3 \times 111 \times I$ $2 \times 3 \times I \times 111$
III o o o o o o	$(111) + (111) + (111) + (111) + (111) + (111)$	$6 \times 1 \times 111 \times I$ $6 \times 1 \times I \times 111$
IV oooooo	$(111+111+111+111+111+111)$	$1 \times 6 \times 111 \times I$ $1 \times 6 \times I \times 111$

Note that the two forms in the Multiplication column for each pattern in the Grouping column are ended with either the $\times 1_{\text{final}}$ or the $\times 1_{\text{middle}}$ element as summarized in Table 4.

Table 4: Two variants of ‘ $\times 1$ ’

Multiplication	$\times 1_{\text{final}}$	$\times 1_{\text{middle}}$
$3 \times 2 \times 111 \times I$	$3 \times 2 \times 111 \times I_{\text{final}}$	
$3 \times 2 \times I \times 111$		$3 \times 2 \times I_{\text{middle}} \times 111$
$2 \times 3 \times 111 \times I$	$2 \times 3 \times 111 \times I_{\text{final}}$	
$2 \times 3 \times I \times 111$		$2 \times 3 \times I_{\text{middle}} \times 111$
$6 \times 1 \times 111 \times I$	$6 \times 1 \times 111 \times I_{\text{final}}$	
$6 \times 1 \times I \times 111$		$6 \times 1 \times I_{\text{middle}} \times 111$
$1 \times 6 \times 111 \times I$	$1 \times 6 \times 111 \times I_{\text{final}}$	
$1 \times 6 \times I \times 111$		$1 \times 6 \times I_{\text{middle}} \times 111$

What are the $\times 1_{\text{final}}$ and the $\times 1_{\text{middle}}$ elements used for? Could they be two variants of the $\times 1$ element as generalized before? If the $\times 1$ element can really be linked to the nominal suffix, which of these two variants is more likely to be the right candidate? Then what would the function of the other variant be? Recall that internal rearrangement of the content of Number 666 has been shown to give rise to different groupings, which is easy to correlate with the parceling or dividing idea in Landman’s and Borer’s sense. Can the remaining $\times 1$ variant be the root of this correlation?

3.2 Roles of ‘ $\times 1$ ’ elements in numbers and groupings

Notice that the $\times 1_{\text{final}}$ -multiplicative forms for Number 666 look similar to $m \times 1$ in that they are all ended with the $\times 1_{\text{final}}$ element repeated as follows:

$$\begin{aligned}
 (29) \quad & 3 \times 2 \times 111 \times 1_{\text{final}} \\
 & 2 \times 3 \times 111 \times 1_{\text{final}} \\
 & 6 \times 1 \times 111 \times 1_{\text{final}} \\
 & 1 \times 6 \times 111 \times 1_{\text{final}}
 \end{aligned}$$

No matter how the pre- $\times 1_{\text{final}}$ elements are arranged, they can unanimously be expressed in the form of 666×1 , i.e.

$$\begin{aligned}
 (30) \quad & 3 \times 2 \times 111 \times 1_{\text{final}} = (3 \times 2 \times 111) \times 1_{\text{final}} = 666 \times 1_{\text{final}} \\
 & 2 \times 3 \times 111 \times 1_{\text{final}} = (2 \times 3 \times 111) \times 1_{\text{final}} = 666 \times 1_{\text{final}} \\
 & 6 \times 1 \times 111 \times 1_{\text{final}} = (6 \times 1 \times 111) \times 1_{\text{final}} = 666 \times 1_{\text{final}} \\
 & 1 \times 6 \times 111 \times 1_{\text{final}} = (1 \times 6 \times 111) \times 1_{\text{final}} = 666 \times 1_{\text{final}}
 \end{aligned}$$

Since $m \times 1$ is argued for number calling in §3.1, the $\times 1_{\text{final}}$ -multiplication form is also considered responsible for number calling. The $\times 1_{\text{final}}$ element ‘locks’ the whole number at the cardinality of six hundred and sixty-six in this case.

How does the $\times 1_{\text{final}}$ element ‘lock’ the number? Let us see what happens inside the calling process. Given that the calling of Number 666 is represented by 666×1 , if 666 in 666×1 is expressed by means of the power of base ten, then

$$\begin{aligned}
 (31) \quad 666 &= 666 \times 1_{\text{final}} = (6 \times 10^2 + 6 \times 10^1 + 6 \times 10^0) \times 1_{\text{final}} \\
 &= 6 \times 10^2 \times 1_{\text{final}} + 6 \times 10^1 \times 1_{\text{final}} + 6 \times 10^0 \times 1_{\text{final}} \\
 &= 6 \times (10^2 \times 1_{\text{final}}) + 6 \times (10^1 \times 1_{\text{final}}) + 6 \times (10^0 \times 1_{\text{final}}) - (a)
 \end{aligned}$$

In each bracket pair of (a) in (31), the ten-based multiple component and the $\times 1_{\text{final}}$ element combine to give rise to the verbal form of the multiple as *bai/hundred*, *shi/ten* and *yi/one* through the derivation in (32).

$$\begin{aligned}
 (32) \quad & 6 \times (10^2 \times 1_{\text{final}}) \quad + \quad 6 \times (10^1 \times 1_{\text{final}}) \quad + \quad 6 \times (10^0 \times 1_{\text{final}}) \\
 \rightarrow & 6 (10^2 - 1_{\text{final}}) \quad \quad 6 (10^1 - 1_{\text{final}}) \quad \quad 6 (10^0 - 1_{\text{final}}) \\
 \rightarrow & 6 (\text{bai}) \quad \quad \quad 6 (\text{shi}) \quad \quad \quad 6 (\emptyset) / *6 (\text{yi}) \\
 \rightarrow & 6 (\text{hundred}) \quad \quad 6 (-\text{ty}) \quad \quad \quad 6 (\emptyset) / *6 (\text{one})
 \end{aligned}$$

What gives us the impression of the $\times 1_{\text{final}}$ element that locks the number is its attachment on each multiple. So let me propose that this $\times 1_{\text{final}}$ element (i.e. the variant of the $\times 1$ element in the final position of the multiplicative representation in number calling) should be present in all multiple portions of all numbers and serve as the input to the language faculty for number coding. It is subsequently realized as the nominal

suffix in Kayne's sense attaching on the multiple portion of each digit in a number, be the suffix overt or covert. For example, the nominal suffix (NS) is actualized as silent in both Chinese and English as in (33). With this $\times 1_{\text{final}}$ element as the origin, it is no wonder why Kayne postulated that the nominal suffix is essential for the multiplication of numbers.

$$\begin{array}{lll}
 (33) \quad 666 & \rightarrow 6 (10^2 - 1_{\text{final}}) & 6 (10^1 - 1_{\text{final}}) \quad 6 (10^0 - 1_{\text{final}}) \\
 & \rightarrow 6 (\text{bai-NS}_{\text{silent}}) & 6 (\text{shi-NS}_{\text{silent}}) \quad 6 (\emptyset\text{-NS}_{\text{silent}}) \\
 & \rightarrow 6 (\text{hundred-NS}_{\text{silent}}) & 6 (\text{-ty-NS}_{\text{silent}}) \quad 6 (\emptyset\text{-NS}_{\text{silent}})
 \end{array}$$

For a single-digit number or the rightmost digit of a number, r , expressed as $r = r \times (10^0 \times 1_{\text{final}})$, since the $\times 1_{\text{final}}$ element has been proposed to realize the nominal suffix, then r is encoded as r -Nominal Suffix. The question left in Kayne (2005) whether a single-digit number, such as *three*, possesses the nominal suffix has received a satisfactory answer here. *Three* is realized as *three*-Nominal Suffix in English where the suffix is silent. In Chinese, the same number is realized as *san*-Nominal Suffix and the suffix is silent too.

But what about the silent $(10^0 \times 1_{\text{final}})$ component of single-digit numbers and the rightmost digit in a number? Suppose in the language faculty this $\times 1_{\text{final}}$ element has to suffix on each ten-based multiple when a number is spelt out. An ad-hoc reason for the asymmetry on the rightmost digit of numbers or single-digit numbers may be posited as the $\times 1_{\text{final}}$ element not being able to attach on the multiple with the power of ten less than one, i.e. 10^0 . So does it follow that this $\times 1_{\text{final}}$ element can be exempted from the requirement for suffixing on the multiple 10^0 or will there be some remedy for this inability?

This puzzle brings us back to another multiplicative representation for groupings discussed in §3.1. If the $\times 1_{\text{final}}$ -form takes care of denoting the cardinality of the number, and is realized as the nominal suffix in number calling, then the $\times 1_{\text{middle}}$ form is left for denoting different groupings, as repeated in Table 5.

Table 5: ' $\times 1_{\text{middle}}$ ' in different groupings.

$3 \times 2 \times 1_{\text{middle}} \times 111$	$(111+111)+(111+111)+(111+111)$	oo oo oo
$2 \times 3 \times 1_{\text{middle}} \times 111$	$(111+111+111+)+(111+111+111)$	ooo ooo
$6 \times 1 \times 1_{\text{middle}} \times 111$	$(111)+(111)+(111)+(111)+(111)+(111)$	o o o o o o
$1 \times 6 \times 1_{\text{middle}} \times 111$	$(111+111+111+111+111+111)$	oooooo

However, the $\times 1_{\text{middle}}$ element does not do the job alone. It embraces another digit as a pack to shape Number 666 in four different ways.

In $3 \times (2 \times 1_{\text{middle}}) \times 111$, $(2 \times 1_{\text{middle}})$ is read as two 111's in a group and there are three such two-111 groups. In other words, the grouping is done via 'grouping by two'. In $2 \times (3 \times 1_{\text{middle}}) \times 111$, $(3 \times 1_{\text{middle}})$ is read as three 111's in a group and there are two such three-111 groups. The grouping is done via 'grouping by three'. In $6 \times (1 \times 1_{\text{middle}}) \times 111$, $(1 \times 1_{\text{middle}})$ is read as one 111 in a group and there are six such one-111 groups via 'grouping by 1'. Finally in $1 \times (6 \times 1_{\text{middle}}) \times 111$, $(6 \times 1_{\text{middle}})$ is read as six 111's in a group and there is one such six-111 group via 'grouping by 6'.

This $\times 1_{\text{middle}}$ element obviously performs the parceling or dividing function and the size of each act is determined by the digit n in the $(n \times 1_{\text{middle}})$ pack. By now Landman's parceling and Borer's dividing ideas are actually two sides of the $\times 1_{\text{middle}}$ coin.

The 'grouping by 1' form represents the individual counting process used in daily life. After parceling or dividing by the $(1 \times 1_{\text{middle}})$ pack, the resultants are counted to the cardinal number six, which is represented by the leftmost digit 6 in the multiplicative representation. Call it the counter. The 'grouping-by-6' resembles the collective counting process, i.e. collectivizing all things into one single group. This group is then counted by the cardinal number one, represented by the leftmost digit 1 in the multiplicative representation. While these two processes are just two instances of how a number is grouped, both of them can be encoded by the $\times 1_{\text{middle}}$ -multiplicative representations as follows:

Individual counting: Grouping by 1 $\rightarrow 6 \times (1 \times 1_{\text{middle}}) \times 111$ o o o o o o
 Collective counting: Grouping by 6 $\rightarrow 1 \times (6 \times 1_{\text{middle}}) \times 111$ oooooo
 Grouping by 3 $\rightarrow 2 \times (3 \times 1_{\text{middle}}) \times 111$ ooo ooo
 Grouping by 2 $\rightarrow 3 \times (2 \times 1_{\text{middle}}) \times 111$ oo oo oo

Similar argument goes for grouping by 3 and 2. In grouping by 3, the counting result of the parcels or dividends is represented by the leftmost digit 2 in the multiplicative representation. So the counter is 2. In grouping by 2, the counter is 3. We have up till now combined the idea of parceling or dividing and counting in terms of the members in a multiplicative representation, summarized in Table 6.

Table 6: Counter and parceler/divider

Counter	Parceler/Divider	Entity for counting
6	$\times (1 \times 1_{\text{middle}}) \times$	111
1	$\times (6 \times 1_{\text{middle}}) \times$	111
2	$\times (3 \times 1_{\text{middle}}) \times$	111
3	$\times (2 \times 1_{\text{middle}}) \times$	111

Let us move on from counting numbers to counting others with the digit n in the $(n \times 1_{\text{middle}})$ pack kept as one. If the entity changes from Number 111 to Number 1, the empty set \emptyset and the object *qiu/ball*, the resultant multiplicative forms are those in Table 7.

Table 7: Other entities for counting

Counter	Parceler/divider	Entity for counting
6	$\times(1 \times 1_{\text{middle}}) \times$	111
6	$\times(1 \times 1_{\text{middle}}) \times$	1
6	$\times(1 \times 1_{\text{middle}}) \times$	\emptyset
6	$\times(1 \times 1_{\text{middle}}) \times$	qiu/ball

Of course, it sounds strange in the mathematical sense how the noun phrase, *qiu/ball*, forms a multiplicative relation with a number since a noun phrase is not a number. However, suppose there is some number feature in a noun phrase. When it is to be checked off in the computation system of language for the purpose of dividing or parceling, the feature undergoes the multiplicative operation.

What happens when all the forms in the above table are encoded in language? Earlier it had been concluded that number calling requires the $\times 1_{\text{final}}$ element to be realized as the nominal suffix. Now we are dealing with how to encode the counting acts. As counting (through parceling or dividing) is based on the $\times 1_{\text{middle}}$ -multiplication form, the coding of counting means the coding of this $\times 1_{\text{middle}}$ element.

3.3 Emergence of classifiers

Recall that there is a headache left by the $\times 1_{\text{final}}$ element which fails to attach itself as the nominal suffix with a sound on single-digit numbers and the rightmost digit of a number in §3.2. This $\times 1_{\text{final}}$ element is proposed to move to the middle position and become the $\times 1_{\text{middle}}$ variant in the multiplicative representation without altering the final numerical product. What has changed is just the order of components inside the representation. Consider the multiplicative form $6 \times (1 \times 1_{\text{middle}}) \times 111$ (from $6 \times (1) \times 111 \times 1_{\text{final}}$). Landing in the middle position, the $\times 1_{\text{middle}}$ variant is claimed to be capable of discharging its original duty in the final position as ' $\times 1_{\text{final}}$ ' by combining with another digit, 1 in this case, to form the $(1 \times 1_{\text{middle}})$ pack. The original function of the $\times 1_{\text{final}}$ element to encode numbers has changed to the function of producing different groupings in the multiplicative representation. At this point, the previous unfulfilled suffixation by the $\times 1_{\text{final}}$ on single and rightmost digits in number calling has been resolved by this $\times 1_{\text{middle}}$ variant in the multiplicative representation.

The need for suffixation of Kayne's nominal suffix by now has motivated the

change of the $\times 1$ element from the final position as $\times 1_{\text{final}}$ for encoding numbers to the middle position as $\times 1_{\text{middle}}$ for encoding groupings in the multiplicative representation. And this middle position paves the way for the parceling and dividing function of the $\times 1_{\text{middle}}$ variant in the $(1 \times 1_{\text{middle}})$ pack.

In the pack, the $\times 1_{\text{middle}}$ acts as a set responsible for dividing or parceling and the remaining '1' as the size of the set. Since the multiplicative operator can be deleted as in $a \times b \times c = abc$, the forms in Column A of Table 8 can change to those in Column B. The $(1_{\text{size}} \times 1_{\text{set}})$ pack is then subsequently represented in the pre-linguistic forms as SET-1 in Column C.

Table 8: Pre-linguistic forms of multiplication

A	B	C
$6 \times (1 \times 1_{\text{middle}}) \times 111$	$6 \times (1_{\text{size}} \times 1_{\text{set}}) \times 111$	6 SET-1 111
$6 \times (1 \times 1_{\text{middle}}) \times 1$	$6 \times (1_{\text{size}} \times 1_{\text{set}}) \times 1$	6 SET-1 1
$6 \times (1 \times 1_{\text{middle}}) \times \emptyset$	$6 \times (1_{\text{size}} \times 1_{\text{set}}) \times \emptyset$	6 SET-1 \emptyset
$6 \times (1 \times 1_{\text{middle}}) \times \text{qiu/ball}$	$6 \times (1_{\text{size}} \times 1_{\text{set}}) \times \text{qiu/ball}$	6 SET-1 qiu/ball

Regarding this set-and-size feature pack, SET-n, the language faculty realizes it with a sound as in Table 9. Call it the classifier.

Table 9: Classifiers in Chinese and English

Number faculty	Interface	Chinese	English
$6 \times (1 \times 1_{\text{middle}}) \times 111$	6 SET-1 111	6 ge 111	6 111's
$6 \times (1 \times 1_{\text{middle}}) \times 1$	6 SET-1 1	6 ge 1	6 1's
$6 \times (1 \times 1_{\text{middle}}) \times \emptyset$	6 SET-1 \emptyset	6 ge \emptyset	6 \emptyset 's
$6 \times (1 \times 1_{\text{middle}}) \times \text{qiu/ball}$	6 SET-1 qiu/ball	6 ge qiu	6 ball's

In Chinese, the dual-featured-classifier is realized as a free morpheme, such as *ge* in this case, and a bound morpheme *'s* on nouns in English. Since both morphemes come from the $(1_{\text{size}} \times 1_{\text{set}})$ pack, they function as a divider or a parceler.⁸ So far not only have I shown how the multiplicative operation derives the equivalence between the

⁸ Because the classifier *ge* as a free morpheme in Chinese comes from the silent $(10^0 \times 1_{\text{final}})$ component of single-digit numbers and the rightmost digit in a number, *ge* is used to name the former as *ge wei shu* (ones position number) and the position of the latter as *ge wei* (ones position) in order to memorize this silent $(10^0 \times 1_{\text{final}})$ combination. Hence the classifier is also employed in the series of multiple names in Chinese as *ge-shi-bai-qian-wan* (ones-tens-hundreds-thousands-tens.of.thousands).

classifier in Chinese and the plural inflection in English (Borer 2005), but I have also demonstrated how the classifier as a parceler in Landman (2004) and a divider in Borer's study is explained together in terms of the multiplicative representation.

While the set feature of the classifier proposal can be easily linked to the idea of dividing or parceling, the size feature has yet been discussed in the literature, which is very important for the present framework.

The size feature of the classifier will influence the choice of classifier items in Chinese and classifier construction in English. If the size feature changes to six, three, and two for *qiu/ball* as below, different classifier structures are realized in Chinese and English as in Table 10.

Table 10: Expansion of classifiers

$1 \times (6 \times 1_{\text{middle}}) \times \text{qiu/ball}$	1 <i>duī qiu</i>	1 pile of balls
$2 \times (3 \times 1_{\text{middle}}) \times \text{qiu/ball}$	2 <i>zǔ qiu</i>	2 groups of balls
$3 \times (2 \times 1_{\text{middle}}) \times \text{qiu/ball}$	3 <i>duì qiu</i>	3 pairs of balls

In Chinese, *duī* (pile) can be used for the $(6 \times 1_{\text{middle}})$ pack, *zǔ* (group) for the $(3 \times 1_{\text{middle}})$ pack and *duì* (pair) for the $(2 \times 1_{\text{middle}})$ pack. Similarly in English the classifier-*of* construction is used instead but with different words. The respective constructions for the three packs are *pile of*, *groups of* and *pairs of*. Not mentioned in Borer (2005), classifiers with different sizes greater than one have been derived similarly through the $(n \times 1_{\text{middle}})$ pack in the multiplicative representation.⁹

The size feature of the classifier is not discussed in the classifier framework of Landman (2004) either. If his idea of *time* as a classifier combines the size feature, the *four times three boys* expression can be accommodated in Borer's or in the traditional nominal structure as [Number *four*] [Divider/Classifier *times-three*] [Noun *boys*]. The SET feature is realized as *times* which performs the function of parceling. And the size of each parcel is determined by the size feature indicated by *three*. So *times three* as a whole is the classifier realized in Divider/Classifier Phrase.

The set-and-size feature pack of the classifier can also illuminate Kayne's idea of the unpronounced SET in these data: *threes/ groups of three/ sets of three*. First, let us add a number in front of each example so that the entity for counting is *three*, i.e. *four*

⁹ Because of this $n \times 1$ structure of the classifier, it is no wonder why there is an association between number and classifier in Chinese (such as the singular classifier – plural classifier contrast and the universal quantification expressed by the classifier-classifier sequence), and why the classifier can serve the function of a syntactic countability marker as noted in Cheng & Sybesma (2005).

threes / four groups of three / four sets of three. *Four threes* has been shown before as in the analysis of *six ones* that the plural -s on *three* is a classifier, dividing tokens of *three* with the size as one due to the $(1 \times 1_{\text{middle}})$ pack as in Table 11.

Table 11: Single digits and classifiers

Four		threes
Four	[Classifier -s]	three
4	SET-1	3
4	$\times(1 \times 1) \times$	3

As for *four groups / sets of three*, if it is assumed that there is one token of *three* in one group or one set, the expressions also exhibit the multiplicative representation as $4 \times (1 \times 1) \times 3$. Since the (1×1) pack is realized as a classifier or SET-1, the *groups of* or *sets of* structure should also be regarded as a classifier, dividing the tokens of *three* by the size of one as in Table 12.

Table 12: Single digits and classifier-of

Four	[Classifier groups of]	three
Four	[Classifier sets of]	three
4	SET-1	3
4	$\times(1 \times 1) \times$	3

The SET idea of Kayne's examples is in fact our classifier with the set-and-size feature pack that groups or sets the quantity of an entity with a certain size feature due to the $(n \times 1)$ pack in the multiplicative representation.

How the structure of multiplication in mathematics is encoded in language has been shown to give rise to the birth of classifiers as a dual-featured category from a certain portion of multiplication. This multiplicative view of the classifier system has clarified the role of the nominal suffix in the multiplicative representation of a number in §3.2, has illustrated how the nominal suffix is related to the emergence of classifiers and has explained how the classifier emerges as a parceler and a divider in §3.3. In particular, the set-and-size feature pack of the classifier proposal has deduced how the classifier system expands and have determined the categorical status of *times three* in *four times three boys*, as well as the syntax of *threes*, *groups of three* and *sets of three*.

To sum up, when the language faculty encodes numbers and groupings, it has to fulfill the conditions of the multiplicative operation of the number faculty. The bridging of these two interfaces triggers the emergence of classifiers. Therefore, the classifier is a

universal category in the syntax of all languages.

4. Implication: from classifiers to natural numbers

If such a multiplication-oriented explanation of the origin of classifiers is virtually necessary for the design of language, then the impact on parametric variation in this vein will be tremendous. Questions—such as why English uses the plural inflection but not a classifier item as in Chinese, or why Cantonese (but not Mandarin) can use definite classifier-noun phrases, or why English has the classifier-*of* construction (but Chinese does not)—will be subjected to examination via the mathematics lens as to how parametric switches are set in accordance with the structure of multiplication. Of course, going in this direction is no easy task, but is a prosperous arena for future research.

Nevertheless, in the closing section of this paper, I try to explore more the number-language crossover by showing how the mathematics-based study of classifiers can in reverse highlight the design of the number faculty.

Along the flow of the argument developed here, I have not only used multiplication to solve the classifier problem, but also exploited the use of the $\times 1$ element. Called the ‘multiplicative identity’ (Dedekind 1948:102) or ‘unity’ (Olmsted 1962:2), this number one does not alter the product of a multiplicative representation as in $m = m \times 1$ on the one hand. But on the other, the element has been treated as an important factor that needs coding in the language faculty for various reasons, such as for nominal suffixation (e.g. 6×1) and for dividing (e.g. $6 \times (1 \times 1) \times 1$). As tokens of the multiplicative identity increase, the language faculty may see the resultants differently despite the overall equivalence in terms of the final product among these:

$$(34) \quad 6 = 6 \times 1 = 6 \times 1 \times 1 = 6 \times 1 \times 1 \times 1 = 6 \times 1 \times 1 \times 1 \times 1 \dots$$

Now just focus on $6 = 6 \times (1 \times 1) \times 1$ and see how the classifier is related to the definition of natural numbers. According to Zermelo (1908, cited in Ebbinghaus (1991: 361)), natural numbers are defined by means of a series of brackets. As one is assumed as the start of the series, successive numbers are determined by adding a new pair of brackets to embed the previous series of sets as in:

$$2 = \{1\}, 3 = \{\{1\}\}, 4 = \{\{\{1\}\}\}, 5 = \{\{\{\{1\}\}\}\}, 6 = \{\{\{\{\{1\}\}\}\}\}, \dots$$

In verbal form, $\{1\}$ is read as ‘the set of 1’ and defined as 2. $\{\{1\}\}$ is read as ‘the set of the set of 1’ and defined as 3 and so on. As for Number 6, the set representation is $\{\{\{\{\{1\}\}\}\}\}$ and is read as ‘the set of the set of the set of the set of the set of 1’. This

kind of set-theoretic definition of natural numbers is based on the pioneering work in Dedekind (1888, cited in Ebbinghaus (1991:356)).¹⁰

In $6 = 6 \times (1 \times 1) \times 1$, since the (1×1) pack is encoded in language as the classifier, $6 \times (1 \times 1) \times 1$ is expressed as '6 CL 1', which means six tokens of one. Given that $6 = \{\{\{\{\{1\}\}\}\}\}$ and $6 = 6 \times (1 \times 1) \times 1 = 6 \text{ CL } 1$, then $\{\{\{\{\{1\}\}\}\}\} = 6 \text{ CL } 1$.

That is to say, in this final relation, the set-theoretic definition of Number 6 on the left side is equivalent to the counting of the tokens of Number 1 up to the cardinality of six through the classifier on the right side. The set-of-set operation in defining numbers has now been linked to the set-and-size feature of classifiers in language.

First, the set-of-set idea of Number 6 can be paraphrased as setting one for five times. In each setting act, the immediately previous set is embedded in a new set. Could this setting act serve the function of dividing, i.e. dividing (the extension of) Number 1 into six tokens? If this cannot be totally false, this dividing sense in terms of putting a number into a set is similar to the sense of the set feature of the classifier for individuation. Future investigation may hinge on whether the use of the set-of-set mechanism in defining a number is really abstracted from the idea of the classifier.

Second, in the set-of-set operation, each successive setting act has to be assumed as setting the previous set by a certain increment and this increment has to be identical throughout, i.e. each successive set being larger than the previous one by the size of one set but not two sets. This identical increment resembles the size of each dividing act by the classifier, whose size feature is responsible for each cutting size. In other words, the size of each setting act in defining natural numbers looks like the size of each division by the classifier. Before we hope to find out an answer, let us endeavor to ask the significance of linking the size of setting a number to the size feature of the classifier in a future agenda.

The spirit of the bulk of this paper assumes that the classifier system in language has an underlying multiplicative structure. Why the classifier emerges is due to the fact that language has to meet the interface conditions of the multiplicative operation in the number faculty. Then as the argument arrives at this section, what is implied after all may be the other way round, which opens up another direction of the interface inquiry. If natural numbers are related to the classifier structure one way or another, could it be possible that some aspect of the number faculty, for instance, the set-theoretic representation of natural numbers, was no longer the cause, but the result instead? If yes, then in Chomsky's words, 'the number faculty is an abstraction from human language' (Botha 2003:58).

¹⁰ The set-theoretic representation can be alternatively expressed in terms of the empty set in Neumann (1923, cited in Ebbinghaus (1991:361)) as: $0 := \emptyset$, $1 := \{\emptyset\}$, $2 := \{\emptyset, \{\emptyset\}\}$, ..., $5 := \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}\}$, $6 := \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset, \{\emptyset\}\}\}\}\}\}$, ...

5. Conclusion

We have gained some understanding of the design of the number faculty because of the multiplicative strategy of studying classifiers adopted in this paper. By making use of the $\times 1$ element, I have first clarified what the nominal suffix in Kayne (2005) actually does in the multiplication representation of a number. Then I have deduced how the suffix is linked to the emergence of classifiers by the movement of the $\times 1$ element inside the representation. Finally the combination of the $\times 1$ element and another number has given rise to the classifier with the set-and-size feature pack as a parceler in Landman (2004) and a divider in Borer (2005). Although there is still a long way to go as for how the interface between the number faculty and the language faculty should be investigated, this paper has illuminated at least the multiplicative operation as the deeply rooted basis of the design of the classifier system in language. Therefore, if the conditions of multiplication need to be met by the language faculty, the classifier should be the universal candidate.

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量詞源於乘法說

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本文從數學裡的乘法運算入手，探討量詞如何在語言裡形成。當中的問題有三：一、Kayne (2005) 所提議的名詞後綴在乘法結構中扮演什麼角色？二、名詞後綴如何跟量詞的形成扯上關係？三、量詞如何在乘法中演進為負責分組、切割的語法範疇 (Landman 2004, Borer 2005)？我們提議數字「一」在數學同語言的界面之間把名詞後綴推導出來，繼而產生量詞及其特徵。最後本建議還跟數字的集合理論作一比較，看看量詞能否把數字的某些特徵也顯映出來。

關鍵詞：量詞，形成，乘法，數字